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DEVELOPMENT OF A MODEL TO INVESTIGATE THE BEHAVIOUR OF THE TILTED UP TYPE CONTROL SYSTEM OF A SMALL WIND TURBINE

Mahinsasa Narayana and Kapila Peiris

National Engineering Research & Development Centre of Sri Lanka, 2P/17B Industrial Estate, Ekala, Ja-Ela, Sri Lanka, Fax: ++ 94-11-5354597,E-mail: narayana@nerdc.lk

ABSTRACT: In various small-scale wind turbines various types of control mechanisms are used to protect the small-scale wind turbines from high wind speeds, i.e. when the wind speed exceeds the rated value. In tilted up type small-scale wind turbines the rotor is free to move around horizontal as well as vertical axes. Therefore when the wind speed exceeds the rated value the rotor tilt upwards and sideways and comes to a stable position with an inclination to the direction of wind and hence controlling the component of the wind responsible for power generation (and for the rotational speed of the rotor). This paper describe, how a mathematical model is developed by using D' Alambert's principal to investigate the complex behavior of the wind turbine during this transient period and the stable position.

1. INTRODUCTION

Main task of wind turbine is converting wind power to electricity. Wind turbine must face it self into the wind and has to adapt to circumstance. It also needs to protect it self from the violence of winds greater than the rated wind speed this must happen automatically.

Larger wind turbines have computer driven control systems, which operate servomotors, hydraulic motors, and all sorts of paraphernalia. Small wind turbines need simple passive controls where possible.

Small-scale wind turbines always use simple control systems, which are almost free of moving parts prone to troubleshooting. Tilt up method is one simple system to control the small-scale wind turbine. Figure 1 shows the kinematic details of the system, the rotor, generator and counter weights are free to rotate about pivot point as well as about vertical axis (i.e. tower axis). Rotating wind rotor is pivoted in vertical plane and it could tilt upward due to drag force of wind rotor. Drag force is depended on the wind speed. This paper describes a theoretical investigation of the control behaviour of a small wind turbine producing in Sri Lanka. Counter balancing weight or spring mechanism is needed to build the restoring moment to hold the rotor to the wind [4]. Then wind rotor is tilted up ward with respect to the wind speed.

2. MOMENTS FROM D'ALAMBERT'S FORCES

By using D'Alambert's principle a dynamic system can be convert to a static system. Then the system can be analysed by using static equilibrium conditions, in this case for each and every point in the system forces of magnitude $\delta m_i f_i$ are applied in the opposite direction to its acceleration. δm_i is mass at the point i and f_i is acceleration at the point i.

Moments (M_0) of D'Alambert forces on the rotor, generator and counter weight system about "O" (Figure 2);

$$M_o = \sum \left(\overline{R} + \overline{r}_i\right) \times \delta m_i \left(\ddot{\overline{R}} + \ddot{\overline{r}}\right) = \sum \delta m_i \left[\left(\overline{R} + \overline{r}_i\right) \times \ddot{\overline{R}} + \left(\overline{R} + \overline{r}_i\right) \times \ddot{\overline{r}}_i\right]$$

$$= \sum \delta m_i \overline{R} \times \ddot{R} + \underbrace{\ddot{\overline{R}} \sum \delta m_i r_i}_{0} + \underbrace{\overline{\mathrm{R}} \times \sum \delta \mathrm{m_i} \ddot{\overline{r}}_i}_{0} + \sum \delta m_i \overline{r}_i \times \ddot{\overline{r}}_i$$

$$M_0 = m\overline{R} \times \ddot{\overline{R}} + \frac{\sum d\left(\overline{r}_i \times \delta m_i \dot{\overline{r}}_i\right)}{dt}$$

Now in practice $\dot{\theta} >> \dot{\alpha}, \dot{\gamma}$. therefore velocities generators and counter weight system could be neglected when compared with the velocities of the rotor. Now $\dot{\bar{R}} << \dot{\bar{r}}_i$, therefore

Velocity at point "i" $= \dot{\bar{R}} + \dot{\bar{r}}_i = \bar{V}_i \quad \therefore |\bar{V}_i| \approx |\dot{\bar{r}}_i| \approx \dot{\theta}|\bar{r}_{1i}|$

$\dot{\theta} =$ Angular speed of the rotor about rotor axis, $\bar{r}_i = \bar{r}_0 + \bar{r}_{1i}$

Now,

$$\sum \bar{r}_i \times \delta m_i \dot{\bar{r}}_i = \sum \delta m_i (\bar{r}_0 + \bar{r}_{1i}) \times \bar{V}_i = \bar{r}_0 \underbrace{\sum \delta m_i \bar{V}_i}_{0} + \sum \delta m_i \bar{r}_{1i} \times \bar{V}_i \text{ then, } \sum \delta m_i \bar{r}_{1i} \times \bar{V}_i = \bar{I}\dot{\theta}$$

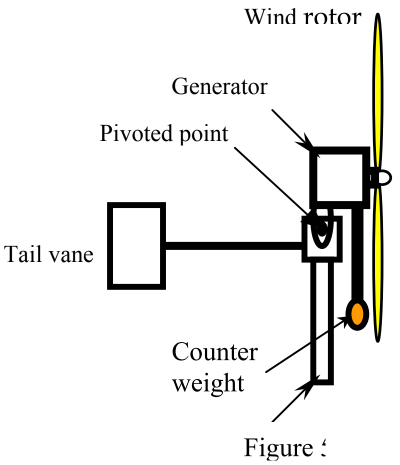

Figure 1: Schematic diagram of tilt up type small wind turbine

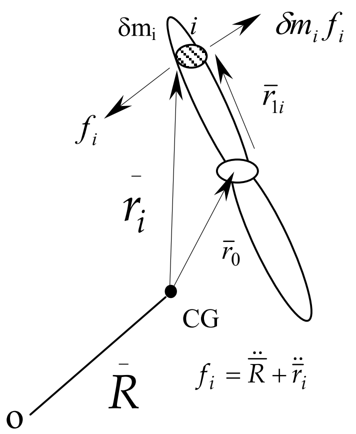

Figure 2: D'Alambert forces on the rotor,generator and counter weight

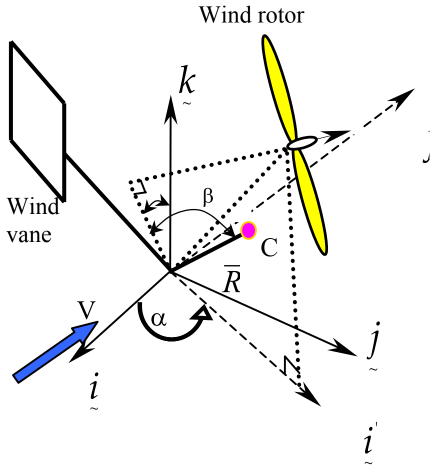

Figure 3: Orientation of wind rotor and wind with respect to co-ordinate system

Where, V_1 – Velocity of wind, CG – Centre of gravity of rotor, generator & counter weight
Therefore;

$$M_0 = m\bar{R} \times \ddot{\bar{R}} + \frac{d(\bar{I}\dot{\theta})}{dt}$$

Where; $|\bar{I}|$ -Moment of inertia of wind rotor about rotor axis, $\dfrac{\bar{I}}{|\bar{I}|}$ -Unit vector along rotor axis,

m –Total mass of the rotor, generator & counter weight,

I_1 -Moment of inertia of tail vane about the $\underset{\sim}{k}$ axis

According to figure 4;

$$\bar{R} = a\sin(\beta-\gamma)\cos\alpha \underline{i} + a\sin(\beta-\gamma)\sin\alpha \underline{j} + a\cos(\beta-\gamma)\underline{k}$$

$$\bar{I} = |\bar{I}|.[\cos\gamma.\cos\alpha.\underline{i} + \cos\gamma.\sin\alpha.\underline{j} + \sin\gamma.\underline{k}]$$

Now, wind rotor, generator, counter weight system & wind vane is free to move about $\underline{k}$ axis.
Therefore,

$$\underline{k} \text{ component of } \left\{ \left[m\bar{R} \times \ddot{\bar{R}} + \frac{d(\bar{I}\dot{\theta})}{dt} \right] - I_1 \ddot{\alpha} \right\} = \left[\begin{array}{l} \underline{k} \text{ component of moment} \\ \text{generated by wind on vane \& rotor} \end{array} \right]$$

Therefore;

$$-m[a\sin(\beta-\gamma)\cos\alpha A - a\sin(\beta-\gamma)\sin\alpha B] - I[\ddot{\theta}\sin\gamma + \dot{\theta}\dot{\gamma}\cos\gamma] - I_1\ddot{\alpha} = M_k \qquad \text{- (1)}$$

$M_k \Rightarrow$ Moment due to aerodynamic forces on rotor and vane about $\underline{k}$ axis

Similarly rotor & counter weight system is free to move about $\underline{j'}$ axis

Therefore,

$$\underline{j}' \text{ component of}\left[m\overline{R}\times\ddot{\overline{R}} + \frac{d(\overline{I}\dot{\theta})}{dt}\right] = \left[\begin{array}{l}\underline{j}\text{ component of moment}\\ \text{generated by wind on vane \& rotor}\end{array}\right].$$

$$\begin{aligned}&\sin\alpha\{a\sin(\beta-\gamma)\sin\alpha[\ddot{\gamma}\sin(\beta-\gamma)-\gamma^2\cos(\beta-\gamma)] - Aa\cos(\beta-\gamma)\}m\\ &-\cos\alpha\{a.\sin(\beta-\gamma)\cos\alpha.a[\ddot{\gamma}\sin(\beta-\gamma)-\gamma^2\cos(\beta-\gamma)] - B.a\cos(\beta-\gamma)\}m\\ &C - mga\sin(\beta-\gamma) = M_{\underline{j}'}\end{aligned} \qquad \text{-(2)}$$

$M_{\underline{j}'} \Rightarrow$ Moment due to aerodynamic forces on rotor about $\underline{j'}$ axis

Where;

$$A = \frac{d^2[a\sin(\beta-\gamma)]}{dt^2}\sin\alpha = a[-\ddot{\gamma}\cos(\beta-\gamma)\sin\alpha - \dot{\gamma}^2\sin(\beta-\gamma)\sin\alpha - \dot{\gamma}\dot{\alpha}\cos(\beta-\gamma)\cos\alpha$$
$$+\ddot{\alpha}\sin(\beta-\gamma)\cos\alpha - \dot{\alpha}\dot{\gamma}\cos(\beta-\gamma)\cos\alpha - \dot{\alpha}^2\sin(\beta-\gamma)\sin\alpha]$$

$$B = \frac{d^2[a\sin(\beta-\gamma)\cos\alpha]}{dt^2} = \alpha[-\ddot{\gamma}\cos(\beta-\gamma)\cos\alpha - \dot{\gamma}^2\sin(\beta-\gamma)\cos\alpha + \dot{\gamma}\dot{\alpha}\sin\alpha\cos(\beta-\gamma)$$
$$-\ddot{\alpha}\sin(\beta-\gamma)\sin\alpha + \dot{\alpha}\dot{\lambda}\cos(\beta-\gamma)\sin\alpha - \dot{\alpha}^2\sin(\beta-\gamma)\cos\alpha]$$

$$C = -|\overline{I}|[\ddot{\theta}\cos\alpha\cos\gamma + \dot{\theta}\dot{\alpha}(-\sin\alpha)\cos\alpha + \dot{\theta}\dot{\gamma}(-\sin\gamma)]\underline{i} + \cos\alpha[\ddot{\theta}\cos\gamma\sin\alpha + \dot{\theta}\dot{\gamma}(-\sin\gamma)\sin\alpha + \dot{\theta}\dot{\alpha}\cos\gamma\cos\alpha]\underline{j}$$

3. AERODYNAMIC FORCES ON THE WIND ROTOR AND VANE

3.1 Aerodynamic forces on wind rotor

An aerodynamic axial thrust force on the rotor is found out by considering the axial momentum of the flow through the rotor. In this study side thrust force is determined by considering wind rotor is a buff body.

$V_2/V_1 = k$ Where; V_1 – Upstream wind speed, V_2 – Down stream wind speed
k -Axial induction factor

The elementary thrust force (dF), can be determined by the general dynamics theory. Consider the axial momentum of the flow through the annulus:

Thrust = (rate of mass flow, m, through the annulus of the thickness dr) x (change in the axial velocity).

Then, axial thrust on the annulus of r_r

$$dF = 2r_r\pi\rho.dr_r V_1^2(1-k^2) \qquad F_t = \frac{1}{2}V_1^2\pi R_r^2\rho C_t \qquad C_t = \frac{2}{R_r}\sum(1-k^2)r_r dr_r$$

Where; R_r –Radius of wind rotor, r_r – Distance from centre of the rotor, dr_r-width of blade element,

ρ -Density of air, Tip speed ratio (λ_0)= $\dot{\theta}R/V_1$

Axial induction factors (*k* values) for each blade elements (annulus) can be determined by comparing the expression for *dF* & *dM* derived by blade elementary theory with that derived by general dynamics [2]. Therefore *k* values for each tip speed ratios (λ_0) can be calculated and then corresponding thrust forces were found.

C_t is the function of λ_0. Then thrust force of each rotor rotational speeds can be calculated for a known tip speed ratio (λ_0).

$$F_t = \frac{1}{2}(V_1 \cos\alpha . \cos\gamma)^2 \pi R_r^2 \rho C_t \quad (3)$$

Side thrust of wind rotor is calculated by considering as drag force of a buff body. Wind velocity component applied on the wind rotor is shown in figure 4. Aerodynamic forces ($F_s \sin\Omega, F_s \cos\Omega \;\&\; F_t$) on the wind rotor are shown in figure 5.

Then, side thrust force is F_s;

$$F_s = \frac{1}{2} C_{ds} V_0^2 \rho A_s \quad (4)$$

Where; F_s -Side thrust, C_{ds}-Drag coefficient for side thrust force of rotor, A_s - Effected rotor area to the side thrust

$$V_0 = V_1\sqrt{(\cos\alpha \sin\gamma)^2 + \sin^2\alpha}$$

For a computation purposes, F_s is divided into two component of $F_s \sin\Omega \;\&\; F_s \cos\Omega$ in the rotor plane.

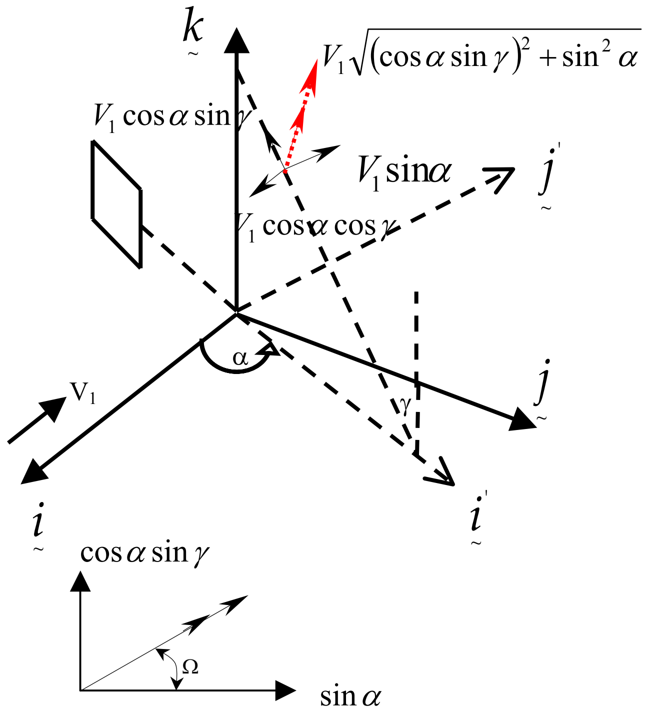

Figure 4: Wind velocity component applied on the wind rotor

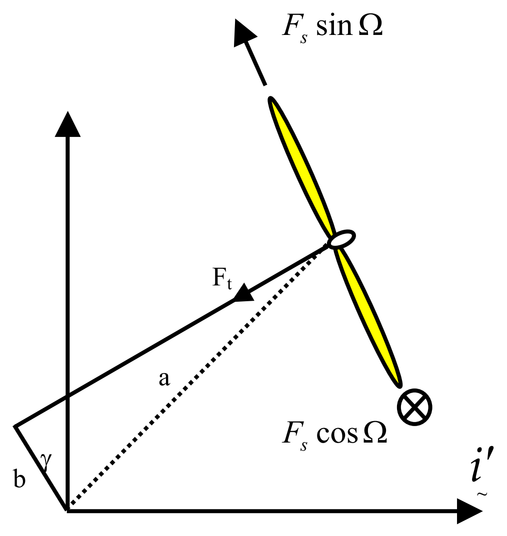

Figure 5: Aerodynamic forces on the wind rotor

3.2 Aerodynamic forces on wind vane

In this wind turbine, a rectangular shape tail vane is used for yawing the wind turbine to correct orientation. Aerodynamic force on the tail vane can be calculated by considering the lift and drag forces. Applied aerodynamic forces on the wind vane are shown in figure 6. Assumed that lift and drag coefficients (C_l and C_d respectively) for tail vane can be approximated by the “flat plate” equations

To determine the aerodynamic forces on the tail vane, relative wind speed to the tail vane should be found. In this calculation, the radial velocity of every point of the tail vane is taken as $\dot{\alpha}L$, since the width of tail vane is relatively small when comparing the length of the vane arm. According to the velocity diagram (figure 7), relative wind speed to the tail vane (V_{tv}) can be derived as follows.

$$V_{vt} = \sqrt{(V_1 - \dot{\alpha}L)^2 + \dot{\alpha}L\cos^2\alpha}$$

According to figure 7; $\psi = \alpha - \tan^{-1}\left[\frac{\dot{\alpha}L\cos\alpha}{V_1 - \dot{\alpha}L\sin\alpha}\right]$,

V_{tv} – Wind speed relative to the tail vane, ψ - Angle of attack to the tail vane

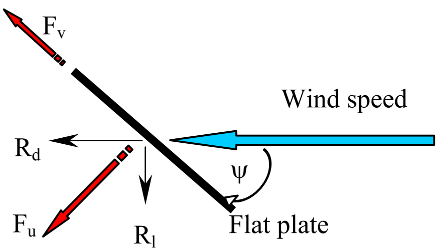

Figure 6: Aerodynamic forces on tail vane

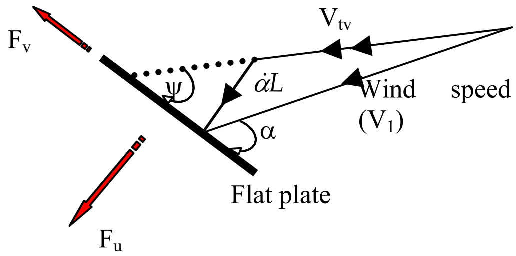

Figure 7: Wind velocity diagram for tail vane

Flat plate equations are: $C_l = 2\sin\psi\cos\psi$ & $C_d = 2\sin^2\psi$

Then lift and drag forces on the wind vane can be calculated as follows;

$$R_l = \frac{1}{2}\rho C_l V_{vt}{}^2 A_v \quad \text{and} \quad R_d = \frac{1}{2}\rho C_d V_{vt} A_v$$

Where; A_V –Area of the tail vane, L –Distance from centroid of the tail vane to the hinge axis.

$$F_u = R_l\cos\psi + R_d\sin\psi \quad \& \quad F_v = R_l\sin\psi - R_d\cos\psi$$

Substitute the solution for C_l, C_d, R_l & R_d in the expression for F_u.

$$F_u = V_{vt}^2\rho A_v\left(\sin\psi\cos^2\psi + \sin^3\psi\right) \qquad (5)$$

4. MOMENTS FROM AERODYNAMIC FORCES ON THE WIND TURBINE

Generated torque by wind rotor due to wind is, depend on the wind speed and rotational speed of the rotor. Torque characteristics (C_m) of a wind rotor can be determined by considering geometrical parameters (blade angles and chord lengths) and characteristics of rotor profiles [3].

$T_W(V, \dot{\theta}) =$ Torque by wind

Resistance torque of generator is, rely on the rotational speed of the generator. Relevant resistant torques for each rotational speed ($\dot{\theta}$) can be found by the torque characteristics of the generator.

$T_L(\dot{\theta}) =$ Torque by load (Generator)

Then; $T_W - T_L = |\bar{I}|\ddot{\theta}$ (6)

Moment due to aerodynamic forces on rotor and vane about $\underline{k}$ axis;

$$M_{\underline{k}} = a\sin(\beta-\gamma)F_s\cos\Omega + F_u L$$

$$M_{\underline{k}} = \frac{1}{2}C_{ds}.V_0^2\rho A_s\cos\Omega a\sin(\beta-\gamma) + LV_{vt}^2\rho A_v\left(\sin\psi\cos^2\psi+\sin^3\psi\right) \quad (7)$$

Substitute the solution for F_s from equation 4 and the solution for F_u from equation 5.
Moment due to aerodynamic forces on rotor about $\underline{j'}$ axis

$$M_{\underline{j'}} = a\cos(\beta-\gamma)F_s\sin\Omega + bF_t$$

Substitute the solution for F_t from equation 3 & solution for F_s from equation 4

$$M_{\underline{j'}} = \frac{1}{2}C_{ds}V_0^2\rho A_s\sin\Omega a\cos(\beta-\gamma) + \frac{1}{2}(V_1\cos\alpha\cos\gamma)^2\pi R_r^2\rho C_t b \quad (8)$$

From equation (1), (2),(6),(7) and (8) it is clear that, if $V_1, \alpha, \dot{\alpha}, \gamma, \dot{\gamma}$ & $\dot{\theta}$ are known at a particular instant (at the time t_0) then for that particular instant $\ddot{\alpha}, \ddot{\gamma}$ & $\ddot{\theta}$ could be determined. Now values of $\alpha, \dot{\alpha}, \gamma, \dot{\gamma}$ & $\dot{\theta}$ after a very small time interval of "δt" could be determine by assuming linear relations within the time interval "δt", now again $, \ddot{\alpha}, \ddot{\gamma}$ & $\ddot{\theta}$ at time $t_0+\delta t$ could be found by equation (1), (2) and (6) with this iterative method it is possible to investigate the variation of kinematic parameters $\alpha, \dot{\alpha}, \ddot{\alpha}, \gamma, \dot{\gamma}, \ddot{\gamma}, \dot{\theta}$ & $\ddot{\theta}$ of the system.

5. CONCLUSIONS

In this analysis, governing equations of the system was derived by using the D'Alambert's principle. Therefore effects such as gyroscopic, centrifugal/centripetal are not relevant in this study. D'Alambert's principle provides a method of analysing a dynamic system with any degree of complex city with a very simple fundamental approach.

Basic external forces applied to the tilt up type wind turbine are from wind. Aerodynamic forces on the wind rotor and tail vane were determined separately. In this study, the rotational speed of the rotor ($\dot{\theta}$) was assumed to be very high compared with the other angular movements ($\dot{\alpha}$ & $\dot{\gamma}$) of the system. In this context, to simplify the solutions, aerodynamic force on the rotor was divided into two categories such as axial thrust and side thrust. The axial thrust force was calculated by considering the wind speed component of the rotor axis and the side thrust force was calculated by considering wind rotor as a buff body. However, the blade elementary theory will give better and comprehensive results although the methodology is complex [1]. The developed model is to be tested with wind tunnel. Then the model could be improved and also the comparison of practical results and predicted value from the model for various iterative time intervals "δt" could be done. Finally with this model it would be possible to optimise the parameters of the system such as dimensions "a", "b" (Figure 5) position & mass of the counter weight in order obtain maximum power output from the wind rotor in varying wind speed condition.

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